Computational Environment for the Study of Optical Waveguides

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Abstract — A notebook in the software Mathematica is developed here for the analysis of planar multilayer dielectric waveguides with the objective of using it as a didactic computational tool, with a possible inclusion in the electrical engineering package library of this same software. The scattering and guiding phenomena in a given structure are analyzed in the notebook, through the use of the programming facilities of the Mathematica software. The user may thus specify the physical and geometrical parameters to be analyzed or make a choice from a model's library that includes periodic structures such as Bragg reflectors and multiple quantum well (MQW) structures.

Index Terms — Dielectric multilayer structure, optical waveguides, periodic structures.

I. INTRODUCTION

The use of educational software [1]–[3] is widely recommended as a complementary tool in applied electromagnetics, and associated disciplines. This results primarily from the fact that these software packages offer a convenient means for mathematical manipulations and for the visualization of the physical phenomena associated with these disciplines. In addition, the high cost of modern laboratories in this area enhances the interest of a more diversified use of software.

The Matlab, Mathcad, Maple, and Mathematica software packages, for example, are widely used in engineering, not only as a teaching but also as a professional tool. These software packages provide a friendly computational environment in which the facilities of numerical, and/or classical algebraic equations solutions combine with the facilities of visualization and document integration. As a result, the difficulty, quite often encountered by undergraduate and graduate students, is partially overcome from the moment that these students are faced with a didactic computational tool of easy comprehension, implemented in an environment familiar to the students.

The main objective here is to propose the implementation of a notebook in the Mathematica software [4], [5] to help the electrical engineering students in the apprenticeship and design of multilayer dielectric waveguides. This study consists of the analysis of scattering, and guiding phenomena in these waveguides. More specifically, it consists of the calculation of the reflection and transmission efficiencies of the waveguide structure and of the observation of the variation of these efficiencies with parameters, such as angle of incidence and wavelength, or in the evaluation of the phase constant and in the visualization of the fields associated with the modes guided by the structure.

The electromagnetic wave propagation study in multilayer dielectric periodic arrays, such as multiple quantum well (MQW) waveguides and Bragg cells, is also treated in the notebook. These structures were included because of the high-frequency selective behavior that they present [6] and for being used in a wide variety of active and passive devices.

The analysis of multilayer dielectric waveguides, periodic or not, is an interesting problem of electromagnetism, not only in a theoretical but also in a practical aspect. This is due to the wide use of these structures in optical devices that extend in operation all the way from microwave to optical frequencies. Lasers, couplers, filters, antireflective surfaces, photodiodes, dividers, and combiners are some examples of these devices.

II. THEORY

The adopted formalism for the study of multilayer dielectric waveguides uses the matrix method described by Born and Wolf [7], and used in [8] and [9]. The waveguides examined are bidimensional and composed of \( k = 2 \) layers, confined between two semiinfinite media: the cover (first layer) and the substrate (last layer), as shown in Fig. 1. The dielectric material of each layer is assumed to be isotropic, linear, homogeneous, nondispersive, and nonmagnetic. It may present gain or loss.

The fields are assumed to be invariant along the \( y \) direction and having a harmonic time dependence of the form \( e^{j\omega t} \), where \( \omega \) is the angular frequency.

With these considerations, the propagating fields in the structure may be decomposed into modes with TE or TM type of polarization.

The tangential components of the electric \( (E_{jk}) \) and magnetic \( (H_{jk}) \) fields, in the \( j \)th layer, may be expressed as a summation of the \( n \)th propagating mode as follows:

\[
E_{jk} = \sum_{n=\pm \infty} A_{kn}^+ e^{-j\gamma_{kn}(z-z_k-1)} + A_{kn}^- e^{+j\gamma_{kn}(z-z_k-1)} \cdot e^{-j\beta_{mn}x}
\]

(1)

\[
H_{jk} = \sum_{n=\pm \infty} T_{kn}^+ e^{-j\gamma_{kn}(z-z_k-1)} - T_{kn}^- e^{+j\gamma_{kn}(z-z_k-1)} \cdot e^{-j\beta_{mn}x}
\]

(2)
The Electromagnetic Behavior of Multilayer Dielectric Waveguides

Introduction

The multilayer dielectric waveguides are widely used in microwave and optical devices, such as semiconductor lasers, filters, couplers, etc.

where $A_{kn}^+$, $A_{kn}^-$ represent the amplitudes of plane waves propagating along the $z$ direction, respectively, as shown in Fig. 1, and $z_k = \sum_{k=1}^{n} h_k$, with $h_1 = 0$. The parameters $h_k$ represent the thicknesses of each layer. The phase constants $\gamma_{kn}$, along the $z$ direction, and $\beta_{kn}$, along the $x$ direction, are related by the following expression:

$$\gamma_{kn} = \sqrt{k_0^2 \epsilon_{rk} - \lambda_{2n}^2},$$

where $k_0 = 2\pi/\lambda_0$ is the free space wave number, $\lambda_0$ is the free space wavelength, and $\epsilon_{rk}$ are the relative permittivities of each layer.

The admittances for the TE modes, along the $z$ direction, are defined by $Y_{kn}^z = \gamma_{kn}/\omega\mu$, where $\mu$ is the permeability of each layer, that, for nonmagnetic dielectrics, will be assumed equal to the free space permeability, $\mu_0$. The expressions for TM modes are obtained following a similar procedure, except that the admittances are given by

$$Y_{kn}^e = \frac{\omega\epsilon_0\epsilon_{rk}}{\gamma_{kn}}.$$

where $\epsilon_0$ is the free space permittivity.

From boundary conditions, the tangential components of the electric and magnetic fields have to be continuous in each dielectric interface between layers. Therefore

$$\begin{bmatrix} E_{tkn} \\ H_{tkn} \end{bmatrix} = \begin{bmatrix} E_{t(k-1)n} \\ H_{tk(k-1)n} \end{bmatrix},$$

(3)

From (3) results that

$$\sum_{n=-\infty}^{\infty} \left[ A_{kn}^+ + A_{kn}^- \right] e^{-j\beta_{kn}x}$$

$$= \sum_{n=-\infty}^{+\infty} \left[ A_{(k-1)n}^+ e^{j\gamma_{kn}h_k} + A_{(k-1)n}^- e^{-j\gamma_{kn}h_k} \right]$$

$$= \sum_{n=-\infty}^{+\infty} Y_{kn}^z \left[ A_{kn}^+ - A_{kn}^- \right] e^{-j\beta_{kn}x}$$

(4)

(5)

Since $e^{-j\beta_{kn}x}$ is a common factor for all terms in (4) and (5), the following expressions are obtained:

$$A_{kn}^+ = \frac{1}{2Y_{kn}^z} \left[ (Y_{kn}^z + Y_{(k-1)n}^-) e^{-j\gamma_{kn}h_k} A_{(k-1)n}^+ \right.$$  

$$+ (Y_{kn}^z - Y_{(k-1)n}^+) e^{j\gamma_{kn}h_k} A_{(k-1)n}^- \right]$$

(6)

$$A_{kn}^- = \frac{1}{2Y_{kn}^z} \left[ (Y_{kn}^z - Y_{(k-1)n}^+) e^{-j\gamma_{kn}h_k} A_{(k-1)n}^+ \right.$$  

$$+ (Y_{kn}^z + Y_{(k-1)n}^-) e^{j\gamma_{kn}h_k} A_{(k-1)n}^- \right]$$

(7)
where \( \gamma_{k} = \frac{\gamma_{k}}{h_{k}} \) represents the phase constant of the \( k \)th propagating mode, along the \( z \) direction, in the \( k \)th layer, normalized with respect to the thickness \( h_{k} \) of this \( k \)th layer.

From (6) and (7) it is possible to relate the amplitude of the propagating mode of any layer with the amplitude of the propagating mode of its preceding one by means of a matrix equation of the form

\[
[A_{kn}] = [M_{(k-1)n}][A_{(k-1)n}]
\]  

(8)

with

\[
(M_{(k-1)n1})_{mn} = \frac{(\gamma_{k}^{1} + \gamma_{(k-1)}^{1})e^{-j\gamma_{kn}}}{2\gamma_{kn}}
\]  

(9)

\[
(M_{(k-1)n2})_{mn} = \frac{(\gamma_{k}^{2} - \gamma_{(k-1)}^{2})e^{+j\gamma_{kn}}}{2\gamma_{kn}}
\]  

(10)

\[
(M_{(k-1)n2})_{mn} = \frac{(\gamma_{k}^{2} - \gamma_{(k-1)}^{2})e^{-j\gamma_{kn}}}{2\gamma_{kn}}
\]  

(11)

\[
(M_{(k-1)n2})_{mn} = \frac{(\gamma_{k}^{2} + \gamma_{(k-1)}^{2})e^{+j\gamma_{kn}}}{2\gamma_{kn}}
\]  

(12)

From (8) the amplitudes of the propagating mode at any interface are related by

\[
[A_{2}] = [M_{1}][A_{1}]
\]  

(13)

\[
[A_{3}] = [M_{2}][A_{2}] = [M_{2}][M_{1}][A_{1}]
\]  

(14)

\[
[A_{kn}] = [M_{(k-1)n}][M_{(k-2)n}]\cdots[M_{2}][M_{1}][A_{1}]
\]  

(15)

By considering \( [Q] = \prod_{k=1}^{K} [M_{k}] \), results

\[
[A_{x}^{+}] = [Q_{11}][Q_{12}][A_{x}^{-}]
\]  

(16)

In addition the reflected mode amplitude is determined from the ratio of the reflected, and incident wave amplitudes. Therefore

\[
[T] = \frac{[A_{x}^{-}]}{[A_{x}^{+}]} = \frac{[Q_{21}][Q_{22}]}{[Q_{11}][Q_{12}][Q_{21}][Q_{22}]-1}.
\]  

(20)

In the second situation, the dielectric multilayers structure is used to guide selected modes. The waveguide is therefore considered as not excited by any external signal (free resonance), that is \([A_{x}^{+}] = [A_{x}^{-}] = [0] \). The real and imaginary phase constants along the longitudinal direction may therefore be determined from this condition. From (16), it requires that

\[
[Q_{22}] = 0.
\]  

(21)

This results in a transcendental equation that may only be solved by means of numerical methods. The analysis of this expression is relevant because it gives the phase constants of the modes that may be guided along the \( x \) direction of the dielectric multilayer structure. The operating frequency, along with the geometrical parameters, and physical characteristics of the waveguide define different solutions for (21).

Various algorithms [8]–[10] have been implemented in order to solve the numerical problems associated with the calculation of the roots of the transcendental equations for optical waveguides. With the use of the Mathematica software, it is possible to observe the graphical display of this equation, and to obtain a point for an initial search, necessary when using Newton’s method or the secant method, as in the case here.

With the solution of (21), the fields distribution in the dielectric multilayer structure may be observed.

III. RESULTS

The notebook, developed in the Mathematica software, analyzes the scattering, and guiding phenomena in simple and periodic structures. Because this is an interactive document, with a didactic objective, it combines text, equations, and graphs in a very convenient way.

The minimum computational requirement specifications to execute the notebook are the same as those required for the Mathematica software, since the notebook was implemented in this software platform.

The basic theory of dielectric multilayer waveguides and the matrix formalism is first presented in the notebook. The user has, therefore, the possibility to familiarize himself with the theory associated with the subject.

In Fig. 1 the notebook may be observed, where the geometry of a dielectric multilayer waveguide is illustrated. The notebook consists of a group of cells that includes text, equations, figures, graphs, and a package of programs. This package may be executed any time the user is interested in observing results from the theoretical study that was done by him in the notebook. Another interesting point is that results from the notebook may also be used in other applications.

As the student executes the cell with the programs he may choose to observe the results from a simple dielectric multilayer structure or from a periodic structure, starting from
a model’s library, that includes MQW waveguides and Bragg cells.

No matter which option is made, the user will always analyze planar waveguides with isotropic, linear, homogeneous, nondispersive, and nonmagnetic layers able to present loss or gain. The physical parameters, such as, the relative permittivity (refraction index) of each layer, and the type of polarization are always requested.

For the analysis of scattering, that is when the structure is used to control the amplitude of the scattering modes, the user may observe the results of the reflection, and transmission efficiencies, or the variation of these efficiencies with the parameters of the problem, particularly, with the angle of incidence in the structure and the wavelength. Fig. 2 shows the dependence of the reflection and transmission efficiencies on the angle of incidence, $\theta$, expressed in degrees.

In the waveguiding analysis, that is, with the dielectric multilayer structure being used to guide selected modes, the user may observe the graph of the dispersion equation, the calculation of the effective index of refraction, and the graphs with the distribution of the tangential components of the electric, and magnetic fields. Examples of graphs with the distribution of the tangential electric field components are shown in Figs. 3 and 4.

In order to solve the transcendental equation, as is the case in (21), the Mathematica software uses Newton’s method or the secant method. The user has, therefore to choose an adequate initial searching point. The observation of the graph for the dispersion characteristics equation is used as an alternative means to choose this initial value.

In Fig. 3, the variation of the normalized electric field intensity, $|E|/|E_{\text{max}}|$, is observed as a function of the normalized thickness, $x/h_{\text{thickness}}$, of an InGaAsP-InP waveguide, operating in the fundamental mode, at 1.55 $\mu$m. The waveguide contains four layers with the following characteristics: $n_1(\text{cover}) = n_4$ (last substrate layer) = 3.169355, $n_2 = 3.252308 - j0.07$. 

![Fig. 2](image)

![Fig. 3](image)
TABLE I
GAIN AND LOSS MODE INDEXES FOR SOME OF THE FIRST FEW TE AND TM FIELDS IN A FIVE-LAYER WAVEGUIDE

<table>
<thead>
<tr>
<th>Mode</th>
<th>Mode Index ($\beta/k_0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TE₀</td>
<td>3.503443329503 + j0.007103009786352</td>
</tr>
<tr>
<td>TE₁</td>
<td>3.33727155849636 + j0.00226891585524791</td>
</tr>
<tr>
<td>TE₂</td>
<td>3.10425142141457 + j0.0013798633975213</td>
</tr>
<tr>
<td>TE₃</td>
<td>2.62813932045903 + j0.00154864433114776</td>
</tr>
<tr>
<td>TE₄</td>
<td>2.24395136260118 + j0.00070837795800883</td>
</tr>
<tr>
<td>TE₅</td>
<td>1.76819096041243 + j0.00135321718385998</td>
</tr>
<tr>
<td>TE₆</td>
<td>1.0746202652578 + j0.002457891743735043</td>
</tr>
<tr>
<td>TM₀</td>
<td>3.49668379589128 + j0.00654398171093878</td>
</tr>
<tr>
<td>TM₁</td>
<td>3.33069711910721 + j0.0000351846222568372</td>
</tr>
<tr>
<td>TM₂</td>
<td>3.2182606966083 - j0.00013470330304956</td>
</tr>
</tbody>
</table>

Fig. 4. Field intensity profile of a five-layer waveguide, with gain and loss.

\[ n_3 = 3.2522398 + j0.07, \quad h_2 = h_3 = 0.5 \mu m. \]

The longitudinal phase constant (phase constant along \( x \) direction) is equal to 3,182,870,182,158,954 + j0,012,733,454,066,1954. The graph was generated by taking the axis origin at the interface between the cover layer and the layer with dielectric loss (layer \( k = 2 \)). The obtained results agree with those from Visser et al. [11], and they show the mode confinement in the layer \( k = 3 \), due to the physical characteristics of the given structure.

From the work of Visser et al. it is also possible to analyze a waveguide with five layers where the cover, and the substrate are free space and the refraction index for the remaining layers are: \( n_2 = n_4 = 3.4 - j0.002, \quad n_3 = 3.6 + j0.01, \quad h_2 = h_4 = 0.6 \mu m, \quad h_3 = 0.4 \mu m, \) for a light wavelength of 1.3 \( \mu m \). The phase constants for some of the first few TE and TM modes of the five layers waveguide are presented in Table I. The user should be alerted that some of the solutions of the transcendental (21) are not trivially obtained and may require a dedicated effort and patience to calculate them.

The normalized field intensities \( |E|/|E_{\text{guide}}| \) as functions of the normalized waveguide thickness \( x/h_{\text{guide}} \) for the TE₀ and TE₃ modes are shown in Fig. 4.

Using a model’s library the student may proceed with the study of periodic structures that are of importance in optoelectronics, and are particularly used in Bragg cells, and in MQW waveguides. The calculation of reflection, and transmission efficiencies in periodic structures is simplified by the matrix method, since the resulting characteristic cell matrix is unimodular [7].

For a given application with Bragg cells, the angle \( \theta \) of the light incident upon the structure may not be arbitrarily chosen such that the user has to look for other varying parameters such as the number \( N \) of cells and the choice of materials to be used. With the use of the matrix formalism the user may therefore observe the variation of the reflection efficiency as a function of the mismatching or of the number of cells in the finite truncated periodic structure.

For the analysis of the Bragg cells, the comparison with the coupled mode theory [12] may be done with the objective of validating the matrix method, and of providing the student an alternate method of analysis.

However, the coupled mode theory ignores final reflections at the structure’s boundary, a price paid for having simpler analytical expressions.

The response of a Bragg reflector is shown in Fig. 5, where the user may visualize the behavior of the reflection efficiency as a function of the incident light free space wavelength \( \lambda_0 \) for different values of number of cells and of dielectric constant ratios \( \varepsilon_{H}/\varepsilon_{L} \), with a relative permittivity of the cover \( \varepsilon_{c} = \varepsilon_{1} = 1 \) (free space) and of the substrate \( \varepsilon_{s} = \varepsilon_{2(N+L)} = 1.52 \).

A rigorous calculation, using the matrix method, for the analysis of the guided wave modes along a structure, such as MQW waveguides, may require a considerable computational effort, due to the extensive mathematical manipulations that are involved. To alleviate this problem approximations have been made such as the one that considers MQW structures as a dielectric slab waveguide, composed of equivalent index of refraction, and thickness, as proposed by Saini and Sharma [13] and Li et al. [14]. The comparison of the results of these approximations is a stimulating problem to be proposed to the students.

IV. CONCLUSIONS

A notebook was implemented in the Mathematica software to be used by electrical engineering students as a didactic computational tool. In this notebook it is possible to analyze scattering and guiding phenomena in simple dielectric multi-layer structures, or in dielectric multilayer periodic structures, that are widely used in the microwave, millimeter-wave, and optical frequency bands.

The Mathematica software was chosen because it has a specific numeric routine to solve transcendental equations, the \textit{FindRoot}, and as is the case with other educational software, because it allows the implementation of specific documents, known as notebooks.
The matrix technique was shown to be efficient for the analysis of planar waveguides with simple multilayers of isotropic, linear, homogeneous, nondispersive, and nonmagnetic dielectrics including the possibility of adding loss or gain.

REFERENCES


Daniela de Oliveira Pereira was born in Belém, PA, Brazil, on January 25, 1976. She received the B.S.E.E. degree from the Federal University of Pará, Brazil in 1998, in electrical engineering. She is presently a Master’s degree student in electrical engineering at the State University of Campinas, Brazil.

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